THE BIAS RATIO – CAN FRAUD BE MODELLED?

Gary van Vuuren PhD, a risk and financial modelling expert asks the question can fraud risk be modelled and therefore predicted. The US$50bn losses expected from the US firm, Madoff Securities (recently exposed as a giant pyramid scheme), are severe and extensive. Astonishingly, the fraud escaped regulatory scrutiny for years. The Securities and Exchange Commission claims that only monthly returns were provided and these did not warrant suspicion. Had the Bias ratio – a metric which augurs potential fraudulent activity – been applied, potential deception would have been signalled after only a few months of monitoring returns. Gary offers a description of the ratio and suggests a practical implementation scheme and then further illustrates this on a South African hedge fund returns.

Introduction

The US hedge fund firm, Madoff Securities LLC, was recently exposed as a giant pyramid scheme and losses derived from its inevitable collapse are now estimated at US$50 billion. Bernie Madoff – the owner of the firm – had provided investors with modest, steady returns, claiming to be generating these by trading in Standard & Poor's 500 Index options. All positions were closed prior to mandatory reporting dates so investors were denied access to the hedge fund holdings. Madoff was a former chairman of NASDAQ Stock Market Inc., well-known, popular and apparently above suspicion: individuals, charities and numerous 'funds of funds' had invested in Madoff Securities hedge funds, amongst them HSBC Holdings PLC and Banco Santander SA. The steady positive returns Madoff offered to his clientele – even in turbulent times – perpetuated the illusion of responsible investing.

The severity and reach of these losses have been disastrous. That the fraud escaped regulatory scrutiny – as well as suspicion, given the level of investor nous – is difficult to comprehend. The US agency responsible for fund oversight (the Securities and Exchange Commission) has claimed that, without significantly more information than the reported monthly fund returns, it could not have detected foul play.

The Bias ratio, introduced in 2008, is a new metric devised to highlight possible fund return manipulation. As such, the ratio may be used as an indicator (but not ultimate proof) of fraudulent activity. Despite its novelty, the Bias ratio would have identified suspicious activity early in the history of the deception (i.e. using relatively few monthly return data): results show that deliberate attempts by Madoff to skew smooth monthly returns would have been exposed after only eight months. The need for wide dissemination of just such early indicators is important, particularly given the fragile nature of the current market which is prone to overreaction to bad news.

The remainder of this review article is structured as follows. Section 2 presents a brief literature overview of the subject of the fraudulent manipulation of returns and other financial statistics. Section 3 outlines the mathematics underlying the Bias ratio and Section 4 then explores some interesting features of the measure. A base case (a normal distribution of returns) is determined and other potential return distributions assessed relative to this base case. The Madoff fund returns are then scrutinised using the Bias ratio and, for completeness, three different sets of hedge fund returns (which each employ a different investment strategy) are examined using the new metric. Section 5 concludes the article.

What the Literature Says

Several studies have reported strong evidence of a positive relation between fund performance and the subsequent flow of investor capital. Berk and Green (2004) interpreted this relationship as the entirely rational response to updated beliefs about fund managers' investment skills. Even after allowing for cumulative returns, investors exhibit an incremental sensitivity to the number of prior monthly losses – in other words, zero was found to be a powerful 'quantitative anchor'. In addition, in order to consistently achieve positive returns (no matter what the economic milieu), many institutional investors pursue 'absolute return' strategies (Waring and Siegel, 2006). Investors are also prone to exaggeration – particularly in the current fragile economic environment – and tend to overreact to bad news (for example, when a negative monthly return is reported, regardless of how small the negative return) lest worse news awaits.

The prevalence of misreporting in the hedge fund industry was investigated by Bollen and Pool (2008) who examined discontinuities in pooled monthly returns.[1] A sharp discontinuity was indeed detected at zero: the number of small gains was significantly greater than expected while the number of small losses was substantially lower. An interpretation of the anomaly was that hedge fund managers distort monthly returns to avoid reporting losses. If this construal were correct, subsequent fund performance should weaken since overstatement must eventually reverse (though this may not necessarily occur in the month immediately subsequent to the

[1] *These returns were those reported to the Center for International Securities and Derivatives Markets (CISDM) database from 1994 to 2005. The CISDM database includes live and defunct hedge funds, funds of funds, CTAs, commodity pool operators, and indices.*

overstatement month). Results from several tests concluded that, indeed, the discontinuity was due to temporarily overstated returns Bollen and Pool (2008). The discontinuity was also found to be prevalent in both live and defunct funds, so it was not simply a reflection of survivorship or backfill bias.

Approximately 10% of fund returns studied were distorted, indicating that overstating returns was a widespread occurrence. Though these small return distortions do not place investors at risk directly, they could indicate a more serious violation of managerial fiduciary duty. Fund net asset values were also overstated when returns were overstated resulting in new investors overpaying for entry to the fund (Getmansky, Lo and Makarov, 2004). If reporting of fund losses is avoided, investors may underestimate hedge fund risks and overestimate managerial performance. As a direct result, investors may allocate more capital to hedge funds than is warranted.

Getmansky, Lo and Makarov (2004) also report that the purposeful smoothing of hedge fund returns biased fund volatility downwards and the Sharpe ratio upwards.

Fund returns were also found to be positively serially correlated. Serial correlation does not necessarily indicate misreporting: positive serial correlation is sometimes recorded when marking to model those funds which are invested in illiquid securities even though there is no intention to deceive. Bollen and Pool (2008) speculated that a fund manager would be more likely to smooth losses than gains, resulting in greater serial correlation when funds perform poorly. Cross-sectional analysis indicates that the propensity for funds to feature conditional serial correlation is positively related to proxies for the risk of capital flight.

Carhart et al. (2002) examined the daily returns of equity mutual funds around quarter (and year)ends and found that funds with the highest year-to-date returns tended to feature larger returns on the last day of a quarter (or a year). These returns were largely reversed the following day. Carhart et al. found that some mutual fund managers temporarily inflated fund asset values by adding illiquid stocks to their positions on the final day of a quarter (or a year). Buying pressure then increased trade prices and the entire position was re-valued upwards. Next-day reversals provided convincing evidence that year-end performance was distorted, since the impact of the trading activity on the last day of the year is only temporary.

Agarwal, Daniel and Naik (2007a) found that average hedge fund returns were higher in December than all other months. The incentive for fund managers to report higher end-of-year returns was measured and the December pattern was found to be more pronounced for managers with higher incentives.

Several studies have also documented evidence of discontinuities in corporate earnings or changes in corporate earnings around zero, including Hayn (1995), Burgstahler and Dichev (1997) and Degeorge, Patel and Zeckhauser (1999). Dechow, Richardson and Tuna (2003) as well as Burgstahler and Dichev (1997) found a discontinuity in the distribution of corporate earnings deflated by market capitalisation.

Large Sharpe ratios have been used to indicate possible fraud (KPMG, 2009:6), Asset purchase prices, returns, periodic growth rates, dividends and other relevant statistics have been employed to accurately assess asset return values, but for illiquid assets these may not be readily available nor reliable. The Bias ratio, introduced by Abdulali (2006), may be used to overcome limitations inherent in other statistics and may also be applied to statistics suspected of manipulation. Some evidence of positive correlation between Sharpe and Bias ratios exists: principal components and neural networks have been used in evaluating this evidence, but these are complex to implement and the output is no less ambiguous than that derived from much simpler techniques (Derrig, 2005).

The Madoff fund has now been exposed as a Ponzi scheme.[2] These offer abnormally high short-term returns to entice new investors. The perpetuation of high returns requires an ever-increasing flow of investor funds in order to maintain the scheme. Ponzi schemes have been responsible for US$ billions (Algo, 2009), but Madoff's deception dwarfs the remainder, as shown in Table 1.

Table 1: Large Ponzi schemes sorted in order of decreasing loss amounts.

Organization	Loss Amount	Settlement Date
Bernard Madoff Investment Services LLC	50,000,000,000	11-Dec-08
Princeton Financial Group	950,000,000	14-Sep-99
Mutual Benefits Corp.	837,000,000	31-Dec-04
Bennett Funding Group Inc.	750,000,000	01-Jan-97
RBG Resources	597,393,000	01-Jun-02
Towers Financial Corporation	500,000,000	31-Dec-93
InverWorld	325,000,000	01-Jan-99
Evergreen Security, Ltd.	214,000,000	31-Dec-01
Mustang Development	139,000,000	31-Mar-95
Gestion Privee Japon	102,000,000	31-Jul-05

Source: Algo FIRST Newsletter.

Technical Details

The Bias ratio operates on return data with mean μ and standard deviation σ. A closed interval $[0.0, +1.0\sigma]$ and a half-open interval $[-1.0\sigma, 0.0)$ is then defined. The fund return in month i is r_i where $1 \le i \le n$ and n is the total number of returns in the data series. The Bias ratio (BR) is then defined as:

$$BR = \frac{\sum_{i=1}^{n} r_i \in [0.0, +1.0\sigma]}{k + \sum_{i=1}^{n} r_i \in [-1.0\sigma, 0.0)} \quad (1)$$

The numerator summation is over the closed interval $[0.0, +1.0\sigma]$ while the denominator summation is over the open half interval $[-1.0\sigma, 0.0)$. The

[2] *Fraudulent investment operation that pay returns to investors from their own invested funds or paid by subsequent investors rather than from profit.*

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small positive constant, k, is included in the formulation to prevent division by zero in cases where there are no returns reported in the interval [–1.0σ, 0.0). In continuous terms, Equation 1 may be stated as follows:

$$BR = \frac{\int_{0}^{+\sigma} r\,dr}{k + \int_{-\sigma}^{0} r\,dr} \qquad (2)$$

The Bias ratio also has the following properties:

1. $0 \leq BR \leq n$,
2. if $r_i \leq 0$, $\forall_i$ then $BR = 0$ and
3. if $r_i > 0$, $r_i > +1\sigma$, $\forall r_i$ then $BR = 0$.

This formulation is easily implemented in spreadsheet software: only return data are required as input.

Data and Results

To understand the operation of the ratio, first consider normally distributed data with μ=0% and σ=1% as shown in Figure 1. A histogram of the data is shown as well as the normal distribution curve on the same x-axis.

The area (using the histogram) over the intervals [0.0,+1.0σ] and [–1.0σ, 0.0) are identical for a normal distribution, hence using Equation 1, BR = 1.0. Return data manipulation should ideally be signalled by several indicators, rather than total reliance upon only a single – potentially fallible – one. There are many ways in which return data may be manipulated, these will be distributionally manifest in prominent ways, i.e. through the mean and the overall shape of the curve, e.g. the skewness and kurtosis. The statistical coefficients of skewness and excess kurtosis (i.e. >3) are thus included in Figure 1 for comparison. For a normal distribution, both of these are 0: the values indicated in Figure 1 above are measured values.

Figure 1: Histogram of normally distributed return data with μ=0% and σ=1% and normal distribution superimposed over the same data bins.

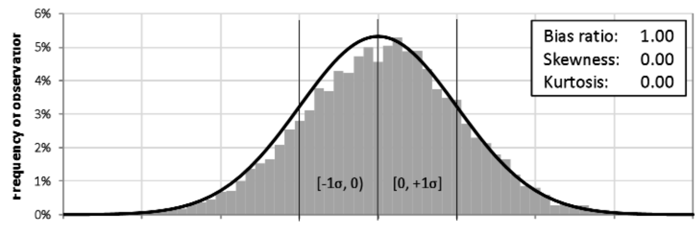


Figure 2: Histogram of normally distributed return data with μ=+1% and σ=1% and normal distribution superimposed over the same data bins.

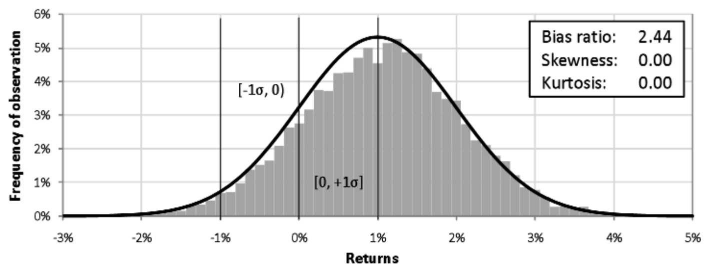
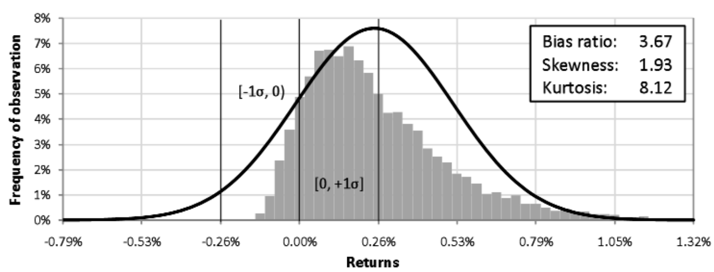


Figure 3: Histogram of lognormally distributed data. The normal distribution is superimposed over the same data bins.



Assume first that the shape of the distribution (in this example, normal) is maintained, but the average return has been altered. There is clearly no incentive to adjust returns such that the new average return is lower than the true mean, hence, any modification is likely to shift the mean in such a way as to only increase it. The situation is shown in Figure 2. The data are normally distributed, but now with μ=+1% and σ=1%. The histogram and normal distribution curve are again shown on the same x-axis.

The areas measured over the requisite intervals are no longer identical. In the example illustrated in Figure 2, Area:[–1.0σ, 0.0) < Area:[0.0, +1.0σ) and Equation 1 gives BR = 2.5. Note that Figure 2 is for illustrative purposes only: there are an infinite number of Bias ratios (all > 0) that could arise from an infinite number of 'mean return adjustments'. It is important to note, however, that a Bias ratio > 1.0 should not automatically trigger suspicion of wrongdoing. Skilled fund managers can easily (and often do) obtain Bias ratios > 1.0 with no deception involved. It is the maintenance of these high Bias ratios for long periods of time as well as the magnitude of the excess above 1.0 that should provoke suspicion.

Assume now that the shape of the distribution is lognormal. Again, there is no incentive to adjust returns such that these returns are negatively skewed (i.e. a long tail to the left of 0%): any fraudulent modifications are likely to adjust returns such that the altered values are positively skewed, i.e. to the right of 0%. The situation is shown in Figure 3. Since the Bias ratio formulation assumes a normal distribution[3] (for the calculation of μ and σ), the 'fit' in Figure 3 is therefore not accurate.

In the particular example shown in Figure 3, Area:[–1.0σ, 0.0) < Area:[0.0, +1.0σ) and Equation 1 gives BR = 3.67. In this case, in addition to the suspiciously high Bias ratio, the high positive skewness of +1.93 (indicating highly skewed returns) and the large excess kurtosis of 8.12 both also warn of potential misrepresentation.

Turning to a practical example – a histogram of the monthly returns from Madoff's Fairfield Sentry hedge fund recorded since 1990 are shown in Figure 4 facing. Superimposed is the best fit normal distribution.

In this case, Area:[–1.0σ, 0.0) << Area:[0.0, +1.0σ) and Equation 1 gives BR = 13.5. This value is particularly high although even a cursory glance at the histogram of returns shows suspiciously few returns below 0% measured over some 19 years. The relatively high positive skewness and excess kurtosis combined with the Bias ratio, should have provoked scepticism. The important conclusion is that these indicators are all measured using only monthly returns.

For completeness, the return series of three different strategies of South African hedge funds were investigated using this analysis. There are different quantities of returns: some were

[3] *The area used in the Bias ratio formulation is, however, empirically derived.*

measured over the period January 2000 to December 2006, others later, but all spanned at least four years ending in December 2006.

Although the skewness coefficient is highest in (b) – the market neutral fund – this value is not disproportionately high. Analysis indicates, however, that all three strategies report high Bias ratios and kurtosis coefficients. In particular, the Bias ratios of funds employing market neutral and fixed income strategies might hint at possible return manipulation. These results are not presented to raise the alarm on South African hedge funds, but rather to illustrate real-life examples of measured Bias ratios and point out the conclusions that may be drawn from high values thereof.

Conclusions

The early detection of fraud – or at the very least – the early signalling of potential fraud is of paramount importance at all times, but particularly in the current economic milieu of falling asset prices, failed banks, reduced lending and broad market uncertainty. The Madoff Securities deception cost investors many US$ billions and, at the time of writing (February 2009), these have not yet necessarily all been disclosed. The need for a simple, effective early warning metric is long overdue. Complex techniques for possible fraud recognition exist, but are usually difficult to implement and exact a heavy resource toll – both in skilled quantitative personnel and in computing requirements. The Bias ratio is a simple, robust technique for evaluating possible deception and may be easily implemented in simple spreadsheet software. Interpreted together with other (standard) statistical coefficients, it could provide the much-needed measurement currently sorely lacking in the market.

Figure 4: Histogram of Madoff's Fairfield Sentry fund return data. The normal distribution is superimposed over the same data bins.

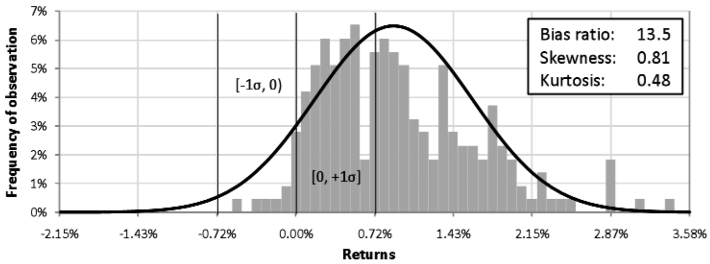
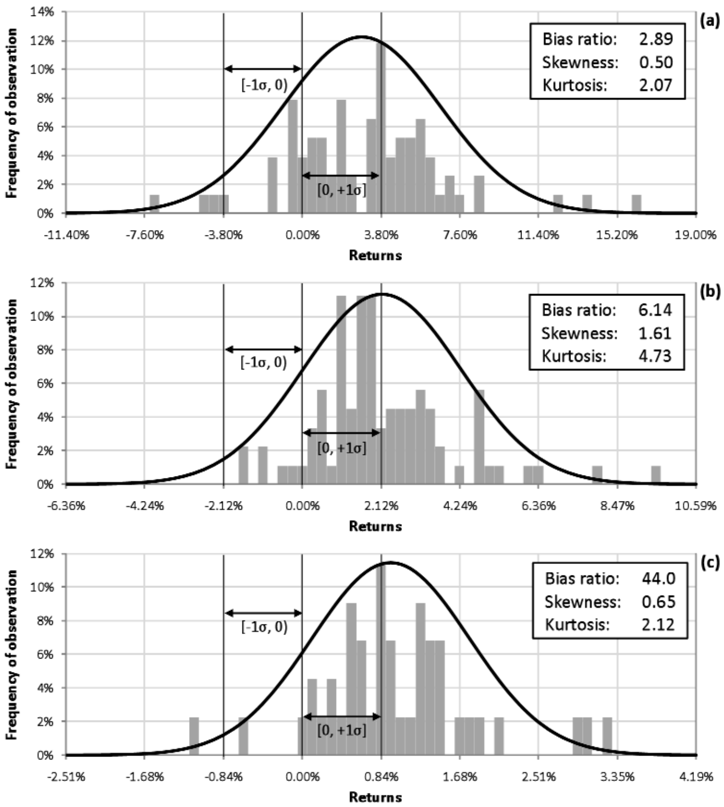


Figure 5: Histogram of three South African hedge fund strategy returns: (a) a long-short equity fund, (b) a market neutral fund and (c) a fixed income hedge fund. The normal distribution is superimposed over the same data bins in each case.



For more on this topic

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